Estimating Marginal Structural Mean Models with Instrumental Variables

Haben Michael, Yifan Cui and Eric Tchetgen Tchetgen

Department of Statistics, The Wharton School, University of Pennsylvania

Abstract

- We consider estimating the parameter β of a Marginal Structural Mean Model:
$$E(Y_{\overline{a}}) = \mu(a;\beta)$$
- Assuming there are no unmeasured confounders ("SRA"), [1] estimates β as the solution to a standard estimating equation suitably re-weighted
- We relax SRA and use IVs to identify and estimate β

Introduction

Notation:

- J time points $j = 1, \dots, J$
- Treatment process $\overline{a} = (a_1, \dots, a_J) \in \{0,1\}^J$
- Counterfactual outcomes $Y_{\overline{a}}$, indexed by treatment
- Observed outcome $Y = \sum_{\overline{a}} 1\{\overline{A} = \overline{a}\} Y_{\overline{a}}$
- Observed covariates $\overline{L} = (L_1, \dots, L_J)$
- Unobserved covariates $\overline{U} = (U_1, \dots, U_J$
- Instrumental variables $\overline{Z} = (Z_1, \dots, Z_J)$

An MSMM is a model on the mean of the potential outcomes:

$$E(Y_{\overline{a}}) = \mu(a;\beta)$$

The parameter β is not in general identified. [1] provides a sufficient condition for identification, the Sequential Randomization Assumption:

$$\overline{Y}_{\overline{a}} \perp\!\!\!\perp A(j) \mid \overline{A}(j-1) = \overline{a}(j-1), \overline{L}(j).$$

Now suppose there is an unmeasured confounder $U_1, \dots, U_J$ of the association between the treatment regime $\overline{A}$ and the potential outcome $Y_{\overline{a}}$, so that SRA is not warranted. We use instrumental variables to identify and estimate the parameter. Informally, an IV is a random variable independent of the unobserved confounder but not independent of the covariate of interest. Equipped with a further assumption on the "compliance type" of the observations, we identify the causal parameter as the solution to a weighted estimating equation, similar to [1]. This result leads to a simple estimator for the causal parameter.

Assumptions

- We relax SRA with "Latent SRA", stating that the potential outcome and treatment are independent provided some unobserved confounder U is observed:

SRA $\Rightarrow$ **Latent SRA**

$$\overline{Y}_{\overline{a}} \perp\!\!\!\perp A(j) \mid \overline{A}(j-1) = \overline{a}(j-1), \overline{L}(j) \Rightarrow \overline{Y}_{\overline{a}} \perp\!\!\!\perp A(j) \mid \overline{A}(j-1) = \overline{a}(j-1), \overline{L}(j), \overline{U}(j), \overline{Z}(j)$$

- Our main assumption is that either compliance type is independent of the unobserved confounder U (**Independent Compliance Type**):

$$\mathbb{E}\left[A(j) | \overline{U}(j), \overline{L}(j), \overline{A}(j-1), \overline{Z}(j-1), Z(j) = 1\right] - \mathbb{E}\left[A(j) | \overline{U}(j), \overline{L}(j), \overline{A}(j-1), \overline{Z}(j-1), Z(j) = 0\right] = \delta_j\left(\overline{L}(j), \overline{A}(j-1), \overline{Z}(j-1)\right)$$

OR that the causal effect is independent of unmeasured confounders: (**Independent Causal Effect**):

$$Y_{\overline{a}_{j-1},1} - Y_{\overline{a}_{j-1},0} \perp\!\!\!\perp \overline{U}_j \mid \overline{L}_j, \overline{A}_{j-1}, \overline{Z}_{j-1}$$

- We also make common IV assumptions:

1. $Z(j) \not\perp\!\!\!\perp A(j) \mid \overline{A}(j-1), \overline{L}(j), \overline{Z}(j-1)$ — **IV Relevance**
2. $\left(\overline{U}, \overline{Y}_{\overline{a}}\right) \perp\!\!\!\perp Z(j) | \overline{A}(j-1) = \overline{a}(j-1), \overline{L}(j), \overline{Z}(j-1)$ — **IV Independence**
3. $0 < \mathbb{P}(Z(j) = 1 | \overline{A}(j-1), \overline{L}(j), \overline{Z}(j-1)) < 1$ — **Positivity**

Weighted Estimating Equation

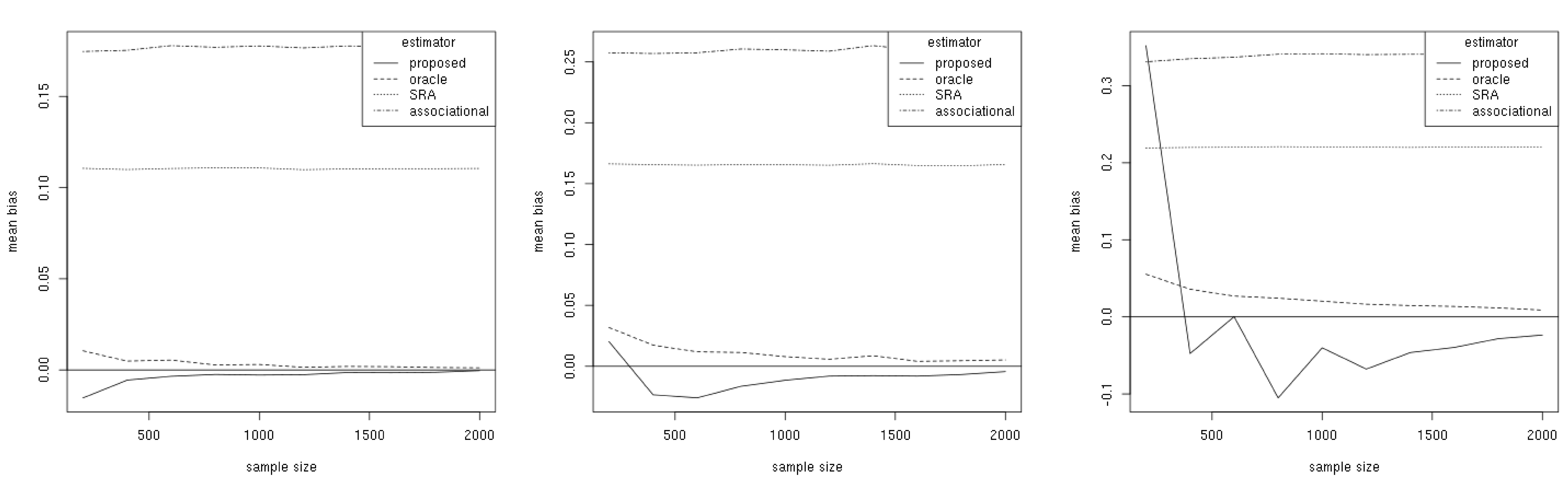
Define weights by

$$\overline{W} = \prod_{j=1}^{J} (-1)^{1-Z_j} \delta_j f_{Z_j}(Z_j \mid \overline{A}_{j-1}, \overline{Z}_{j-1} \overline{L}_{j-1}).$$

Let h denote a vector-valued function of $\overline{A}$ of the same dimension as β. Under the above assumptions,

$$\mathbb{E}\left(h(\overline{A})(Y - \mu_\beta(\overline{A}))/\overline{W}\right) = \sum_{\overline{a}} h(\overline{a})\left(\mathbb{E}(Y_{\overline{a}}) - \mu_\beta(\overline{a})\right)(-1)^{J - \sum_j a_j} = 0$$

Simulation



J=2 J=3 J=4

Mean bias versus sample size of the weighted estimator, for J=2, 3, and 4, time points, compared with oracle (weights including observed and unobserved confounders), SRA (weights including observed confounders), and associational (no weighting) estimators.

Additional Information

- The bootstrap or sandwich variance estimate may be used to carry out inference
- Straightforward extension to discrete-valued treatments A under the "Independent Compliance Assumption", though not under the "Independent Causal Effect" assumption
- The terms δ_j the density f_Z require $\sqrt{n}$-consistent estimation
- Weights may be "stabilized" to the extent that the terms δ_j depend on the treatment process $\overline{A}$, similar to IPW stabilization [1]

See the technical report [2] for the general case, covering any Marginal Structural Model, i.e., any restriction on the distribution of the potential outcome $Y_{\overline{a}}$, including failure times. A semiparametric efficient and multiply robust estimator is also provided there.

References

[1] James Robins.
Marginal Structural Models.
In *1997 Proceedings of the American Statistical Association*, pages 1–10 of 1998 Section on Bayesian Statistical Science, 1997.

[2] E. J Tchetgen Tchetgen, H. Michael, and Y. Cui.
Marginal Structural Models for Time-varying Endogenous Treatments: A Time-Varying Instrumental Variable Approach.
ArXiv e-prints, September 2018.

[3] Haben Michael, Yifan Cui, and Eric J. Tchetgen Tchetgen.
A simple weighted approach for instrumental variable estimation of marginal strucural mean models.
In progress, 2018.

[4] Linbo Wang and Eric Tchetgen Tchetgen.
Bounded, efficient and multiply robust estimation of average treatment effects using instrumental variables.
Journal of the Royal Statistical Society: Series B (Statistical Methodology), 80(3):531–550, 2018.

Contact Information

- Web: https://github.com/haben-michael
- Email: haben@wharton.upenn.edu